

The ground state problem is to find the minimum energy state of a system. This is often done by minimizing the energy function $E(\mathbf{x})$ over the space of possible states \mathbf{x} . In the case of a spin system, the energy function can be written as $E(\mathbf{x}) = \sum_i h_i s_i + \sum_{i < j} J_{ij} s_i s_j$, where $s_i \in \{-1, 1\}$ are the spin variables. This is a Quadratic Unconstrained Binary Optimization (QUBO) problem. The energy function can also be written as $E(\mathbf{x}) = \sum_i Q_{ii} x_i + \sum_{i < j} Q_{ij} x_i x_j$, where $x_i \in \{0, 1\}$ are the binary variables. The ground state problem is then to find the minimum energy state \mathbf{x}^* such that $E(\mathbf{x}^*) \leq E(\mathbf{x})$ for all \mathbf{x} . This is often done by using a quantum annealing algorithm, such as the Quantum Annealer Eigensolver Algorithm (QAEA). The QAEA algorithm is based on the idea of adiabatic evolution. It starts with a simple Hamiltonian H_0 whose ground state is known, and then slowly evolves it to the target Hamiltonian H_1 . If the evolution is slow enough, the system will remain in the ground state of H_0 and thus reach the ground state of H_1 . The QAEA algorithm is implemented on a quantum annealer, such as the D-Wave quantum annealer. The D-Wave quantum annealer is a quantum annealing device that uses superconducting qubits to solve optimization problems. It is based on the idea of quantum annealing, which is a quantum version of simulated annealing. The D-Wave quantum annealer is currently the only quantum annealing device available commercially.

2. QUANTUM ANNEALER EIGENSOLVER ALGORITHM

2.1. Mapping of a Ground State Problem to a QUBO Problem. The ground state problem is to find the minimum energy state of a system. This is often done by minimizing the energy function $E(\mathbf{x})$ over the space of possible states \mathbf{x} . In the case of a spin system, the energy function can be written as $E(\mathbf{x}) = \sum_i h_i s_i + \sum_{i < j} J_{ij} s_i s_j$, where $s_i \in \{-1, 1\}$ are the spin variables. This is a Quadratic Unconstrained Binary Optimization (QUBO) problem. The energy function can also be written as $E(\mathbf{x}) = \sum_i Q_{ii} x_i + \sum_{i < j} Q_{ij} x_i x_j$, where $x_i \in \{0, 1\}$ are the binary variables. The ground state problem is then to find the minimum energy state \mathbf{x}^* such that $E(\mathbf{x}^*) \leq E(\mathbf{x})$ for all \mathbf{x} . This is often done by using a quantum annealing algorithm, such as the Quantum Annealer Eigensolver Algorithm (QAEA). The QAEA algorithm is based on the idea of adiabatic evolution. It starts with a simple Hamiltonian H_0 whose ground state is known, and then slowly evolves it to the target Hamiltonian H_1 . If the evolution is slow enough, the system will remain in the ground state of H_0 and thus reach the ground state of H_1 . The QAEA algorithm is implemented on a quantum annealer, such as the D-Wave quantum annealer. The D-Wave quantum annealer is a quantum annealing device that uses superconducting qubits to solve optimization problems. It is based on the idea of quantum annealing, which is a quantum version of simulated annealing. The D-Wave quantum annealer is currently the only quantum annealing device available commercially.

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Notes

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